Non-Gaussian bias from peak-background split

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VD, Crocce, Scoccimarro, Sheth (2010)
VD, Jeong, Schmidt (in prep)

Peak-background split (I)

- Local (Lagrangian) bias:

$$\langle \delta_g | \delta_l \rangle(\mathbf{x}) \equiv \frac{n_g(\mathbf{x}|\delta_l)}{\bar{n}_g} - 1 = \sum_{N=0}^{\infty} \frac{b_N(R_s)}{N!} \left[\delta(R_l, \mathbf{x})\right]^N$$

- Take the limit $R_l >> R_s$:

$$b_N = \left(-\frac{1}{\sigma_{0s}}\right)^N \bar{n}^{-1} \frac{\partial^N \bar{n}}{\partial \nu^N}, \quad \nu \equiv \delta_c / \sigma_0(R_s)$$

(Bardeen et. al. 1986; Cole & Kaiser 1989; Mo & White 1996; Sheth & Tormen 1999;...)

The peaks model

- Galaxies/halos trace local density maxima of the evolved mass distribution -> include the peak constraint

- Lagrangian bias -> study local maxima of the initial, Gaussian density field

(Peacock & Heavens 1985; Bardeen et al 1986; ...)

Peaks 2-point correlation

$$\xi_{\rm pk}(\nu, R_s, r) = \left(\tilde{\rm b}_I^2 \xi_s\right)(r) + \frac{1}{2}\left(\xi_s \tilde{\rm b}_{II}^2 \xi_s\right)(r)$$

$$+ \text{ other 2nd order terms } + \ \ldots$$

where

$$\tilde{\rm b}_I(\nu, R_s, k) = \tilde{b}_{10}(\nu, R_s) + \tilde{b}_{01}(\nu, R_s) k^2$$

$$\tilde{\rm b}_{II}(\nu, R_s, k_1, k_2) = \tilde{b}_{20}(\nu, R_s) + \tilde{b}_{11}(\nu, R_s)\left(k_1^2 + k_2^2\right) + \tilde{b}_{02} k_1^2 k_2^2$$

(VD, Crocce, Scoccimarro, Sheth 2010)

Peak-background split (II)

- The scale-independent pieces are the peak-background split bias, i.e.

$$\tilde{b}_{N0}(\nu, R_s) = \left(-\frac{1}{\sigma_{0s}}\right)^N \bar{n}_{\rm pk}^{-1}\frac{\partial^N \bar{n}_{\rm pk}}{\partial \nu^N} \equiv b_N(\nu, R_s)$$

- However, derivatives of the peaks number density cannot produce the k-dependent bias terms like

$$\tilde{b}_{01},\ \tilde{b}_{11}\ \text{etc.}$$

Peak-background split (III)

- Consider another implementation of the peak-background split, in which the dependence of the mass function on the background overdensity is derived explicitly

Conditional peaks number density

$$\bar{n}_{\rm pk}(\nu, R_s|\delta_l, R_l) = \frac{G_0(\tilde{\gamma}_1, \tilde{\gamma}_1\tilde{\nu})}{(2\pi)^{3/2}R_1^3}\frac{\exp\left[-\left(\nu-\epsilon\nu_l\right)^2/2\left(1-\epsilon^2\right)\right]}{\sqrt{2\pi\left(1-\epsilon^2\right)}}$$

where

$$\nu_l \equiv \frac{\delta_l}{\sigma_{0l}}, \quad \langle\nu\nu_l\rangle \equiv \epsilon = \frac{\sigma_{0\times}^2}{\sigma_{0s}\sigma_{0l}}, \quad \langle u\nu_l\rangle \equiv \gamma_1\epsilon r, \quad r \equiv \frac{\left\langle k^2\right\rangle_\times}{\left\langle k^2\right\rangle_s} = \frac{\sigma_{1\times}^2/\sigma_{1s}^2}{\sigma_{0\times}^2/\sigma_{0s}^2}$$

$$\langle u|\nu,\nu_l\rangle \equiv \tilde{\gamma}_1\tilde{\nu} = \gamma_1\nu\left(\frac{1-\epsilon^2 r}{1-\epsilon^2}\right) - \gamma_1\left(\frac{1-r}{1-\epsilon^2}\right)\epsilon\nu_l$$

$$\mathrm{Var}(u|\nu,\nu_l) \equiv 1-\tilde{\gamma}_1^2 = 1-\gamma_1^2\left[1+\epsilon^2\frac{(1-r)^2}{1-\epsilon^2}\right]$$

and

$$\sigma_{n\times}^2 \equiv \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)}P_\delta(k)W(kR_s)W(kR_l)$$

- Take ε→0, retain the *r* dependence and expand

$$\langle \delta_{\rm pk} | \delta_l \rangle \equiv \frac{\bar{n}_{\rm pk}(\nu, R_s | \delta_l, R_l)}{\bar{n}_{\rm pk}(\nu, R_s)} - 1$$

in powers of δ_l:

$$\begin{aligned}\langle \delta_{\rm pk} | \delta_l \rangle = & \left[\frac{\sigma_{0\times}^2}{\sigma_{0l}^2} \tilde{b}_{10} + \frac{\sigma_{1\times}^2}{\sigma_{0l}^2} \tilde{b}_{01} \right] \delta_l \\ & + \frac{1}{2} \left[\left(\frac{\sigma_{0\times}^2}{\sigma_{0l}^2} \right)^2 \tilde{b}_{20} + 2 \left(\frac{\sigma_{0\times}^2}{\sigma_{0l}^2} \right)^2 \left(\frac{\sigma_{1\times}^2}{\sigma_{0l}^2} \right)^2 \tilde{b}_{11} + \left(\frac{\sigma_{1\times}^2}{\sigma_{0l}^2} \right)^2 \tilde{b}_{02} \right] \delta_l^2 \\ & + \ \ldots \end{aligned}$$

- Cross-correlate with δ_l:

$$\langle \delta_{\rm pk} \delta_l \rangle = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 \left(\tilde{b}_{10} + \tilde{b}_{01} k^2 \right) P_\delta(k) W(kR_s) W(kR_l) + \cdots$$

Non-Gaussian halo bias

- Can we derive the non-Gaussian bias factors from the (non-Gaussian) conditional mass function ?

(VD, Jeong, Schmidt, in prep)

- The goal is to expand

$$\langle\delta_{\rm pk}|\delta_l\rangle_{\rm NG} \equiv \frac{\bar{n}_{\rm NG}(\nu, R_s|\delta_l, R_l)}{\bar{n}_{\rm NG}(\nu, R_s)} - 1 = \frac{\frac{d}{dM_s}\int_\nu^\infty dx\, P_{\rm NG}(x, R_s|\delta_l, R_l)}{\frac{d}{dM_s}\int_\nu^\infty dx\, P_{\rm NG}(x, R_s)} - 1$$

in powers of δ_l

- Use the relation

$$P_{\rm NG}(\mathbf{y}) = \exp\left[\sum_{N=3}^{\infty} \frac{(-1)^N}{N!} \sum_{\mu_1 \cdots \mu_N}^{N} \langle y_{\mu_1} \cdots y_{\mu_N} \rangle_c \frac{\partial^N}{\partial y_{\mu_1} \cdots \partial y_{\mu_N}}\right] P_{\rm G}(\mathbf{y})$$

$$P_{\rm NG}(\nu, R_s) \text{ depends on } \left\langle \nu^N \right\rangle_c$$

$$P_{\rm NG}(\nu, R_s|\delta_l, R_l) \text{ depends on } \left\langle \nu^m \nu_l^{N-m} \right\rangle_c, \ \nu_l = \frac{\delta_l}{\sigma_{0l}}$$

- For a Press-Schechter mass function:

$$b_N(\nu) = \frac{1}{\sigma_{0s}^N} \frac{H_{N+1}(\nu)}{\nu}$$

- Define the quantity

$$\Sigma_{\times}^2 \equiv \frac{1}{2\pi^2}\int_0^{\infty} dk\, k^2 P_{\phi}(k)\mathcal{M}(k,R_s)\mathcal{M}(k,R_l)\mathcal{S}(k,R_s,R_l)$$

$$\delta_R(\mathbf{k}) = \mathcal{M}(k,R)\Phi(\mathbf{k})$$

For instance: $\mathcal{S}(k,R_s,R_l) = k^2 \Rightarrow \Sigma_{\times}^2 \equiv \sigma_{1\times}^2$

$$\mathcal{S}(k,R_s,R_l) = \mathcal{M}(k,R_s)^{-1}\int \frac{d^3q}{(2\pi)^3}\mathcal{M}(q,R_s)\mathcal{M}(|k\hat{\mathbf{z}}+\mathbf{q}|,R_s)\frac{\xi_{\Phi}^{(3)}\left(k\hat{\mathbf{z}},\mathbf{q},-k\hat{\mathbf{z}}-\mathbf{q}\right)}{P_{\phi}(k)}$$

$$\Rightarrow \Sigma_{\times}^2 \equiv \sigma_{0s}^2\sigma_{0l}\left\langle \nu^2\nu_l\right\rangle$$

- On taking the limit $R_l \to \infty$, only the terms with $S=S(k,R_s)$ survive

- Assuming the primordial *N*-point function dominates,

$$\langle \delta_{\rm pk} | \delta_l \rangle_{\rm NG} \approx \left(\frac{\sigma_{0\times}^2}{\sigma_{0l}^2} \right) b_I \delta_l + \left(\frac{\sigma_{0\times}^2}{\sigma_{0l}^2} \right) g \Big[\left\langle \nu^N \right\rangle_c', \left\langle \nu^N \right\rangle_c \Big] \delta_l \\ + \frac{1}{(N-1)!} \left[\left\langle \nu^{N-1} \nu_l \right\rangle_c' H_{N-2}(\nu) + \left\langle \nu^{N-1} \nu_l \right\rangle_c H_{N-1}(\nu) \right] \frac{\delta_l}{\sigma_{0l}}$$

- The scale-dependent bias correction Δb_K can be read off from the terms involving

$$\left\langle \nu^{N-1} \nu_l \right\rangle_c \equiv \Sigma_\times^2 / \left(\sigma_{0s}^{N-1} \sigma_{0l} \right)$$

and its derivative

General formula

$$\Delta \mathrm{b_I}(k) = \frac{4}{(N-1)!}\left\{b_{N-2}\delta_c + b_{N-3}\left[3 - N + \frac{\partial \mathcal{F}_s^{(N)}(k)}{\partial \ln \sigma_{0s}}\right]\right\}\mathcal{F}_s^{(N)}(k)\mathcal{M}(k, R_s)^{-1}$$

where

$$\mathcal{F}_s^{(N)}(k) = \frac{1}{4\sigma_{0s}^2 P_\phi(k)}\left\{\prod_{i=1}^{N-2}\int \frac{d^3k_i}{(2\pi)^3}\mathcal{M}(k_i, R_s)\right\}\mathcal{M}(q, R_s)\xi_\Phi^{(N)}(\mathbf{k}_1, \cdots, \mathbf{k}_{N-2}, \mathbf{q}, \mathbf{k})$$

e.g., for g_{NL} ϕ^3(in low-*k* limit):

$$\Delta \mathrm{b_I}(k) = \frac{1}{2} g_{\mathrm{NL}} \sigma_{0s}^2 S_{s,\mathrm{loc}}^{(3)}\left[b_2\delta_c + \left(1 + \frac{\partial \ln S_{s,\mathrm{loc}}^{(3)}}{\partial \ln \sigma_{0s}}\right) b_1\right]\mathcal{M}(k, R_s)^{-1}$$

Summary

- Scale-dependent Gaussian or non-Gaussian bias factors can be computed upon formulating peak-background split in terms of conditional mass functions